The Relation between the Evolution of Heat and the Supply of Energy during the Passage of an Electric Discharge through Hydrogen.

By J. Keith Roberts, M.Sc., Trinity College, Cambridge; Exhibition of 1851 Scholar of the University of Melbourne.

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I. Introduction.

In this paper an account is given of measurements of the heat evolved by the passage of a known current under a known potential through a hydrogen discharge tube. The experiment was suggested by Sir Ernest Rutherford as a possible test of the hypothesis in which he supposed that a much closer combination between an electron and a hydrogen nucleus than that existing in the hydrogen atom might occur under suitable conditions.* The experiments of Aston, which show that the masses of all atoms except hydrogen are whole numbers, indicate that the close binding of electrons and positive nuclei causes a decrease of mass, owing to the interaction of the fields. Such a decrease of mass might be expected to occur if the supposed close combination of a single hydrogen nucleus and an electron took place. We should expect this decrease of mass to be accompanied by an evolution of energy, since a mass m has energy $\frac{1}{2}mc^2$ associated with it, c being the velocity of light.

In order to account for the fact that the energy radiated by giant stars is too great to be supplied by the loss of gravitational potential energy by contraction, it has been suggested† that atomic changes of this nature occur in these stars. If 1 per cent. of the mass of a giant star were converted from hydrogen to helium the energy evolved would be sufficient to supply the star with energy for $1{\cdot}5 \times 10^7$ years.

In such a star violent collisions will occur between electrons and hydrogen nuclei owing to the high temperature. Such collisions will also occur in a discharge tube containing hydrogen. If in this case these collisions give rise to the changes mentioned, it might be expected that more heat would be evolved from the discharge tube than could be accounted for by the supply of electrical energy as measured by the product of the current and the potential difference.

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* 'Roy. Soc. Proc.,' A, vol. 97, p. 396 (1920).

† Perrin, 'Annal. de Phys.,' vol. 11, p. 90 (1919); Sommerfeld, 'Atombau und Spektral-Linien,' p. 538 (1919); Eddington, 'Nature,' Sept. 2, p. 19 (1920).

To test whether this was the case, a discharge tube was placed in a calorimeter and a comparison made between the heat produced and the energy supplied. It was found that these two quantities agreed to less than $\frac{1}{2}$ per cent., which was within the error of the experiment.

To appreciate the meaning of this negative result, it is necessary to consider in such detail as is possible the conditions prevailing in the discharge tube. The energy of translation of an electron after falling through one volt is $1{\cdot}59 \times 10^{-12}$ ergs, since its charge is $4{\cdot}77 \times 10^{-10}$ e.s.u., and one volt is equal to 1/300 e.s.u. The mean energy of translation of a molecule of a gas at absolute temperature T is equal to 3/2 RT, where $R = 1{\cdot}37 \times 10^{-16}$. The electron after falling through one volt therefore has translational energy equal to the mean translational energy of a gas molecule at temperature T given by

$$T = \frac{2 \times 1{\cdot}59 \times 10^{-12}}{3 \times 1{\cdot}37 \times 10^{-16}} = 7{,}730^\circ.$$

We must now consider the mean free path of the electron. To do this an estimate must be made of the radius of the sphere of action of the hydrogen atom. This will be of the order 10^{-8} cm., which is sufficiently accurate for the present purpose. Taking the number of atoms of hydrogen in a cubic centimetre of gas at atmospheric pressure as $2 \times 2{\cdot}7 \times 10^{19}$, the mean free path, λ, of an electron which is both so small that it can be regarded as a point, and is moving so fast that the hydrogen molecules may be assumed at rest, is given by

$$\lambda = \frac{760}{10^{-16} \times 2 \times 2{\cdot}7 \times 10^{19} \times p} = \frac{0{\cdot}04}{p},$$

where p is the pressure in millimetres of mercury.

The conditions under which the discharge tube was run were such that there was no positive column. In this case nearly the whole drop of potential occurs in the cathode dark space, and the potential gradient is uniform within that region.* If we assume that at each collision an electron loses the whole of its energy, the minimum value possible for the average energy with which an electron leaves the dark space can be obtained by finding the potential drop in a free path and calculating the energy gained by an electron in falling through this potential.

We cannot say what happens to the electrons after they leave the dark space. The only information that this calculation gives us is that electrons are projected from the dark space into the gas in the remainder of the tube, with energies of this order and greater energies. In the experiments carried out the conditions lay between the limits indicated in the following table :—

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* Aston, 'Roy. Soc. Proc.,' A, vol. 84, p. 530 (1911).

Pressure (mm. of mercury).	Length of dark space (cm. L.).	Mean free path of electron (calculated) (cm. λ).	Number of free paths in dark space (L/λ = n).	Total voltage (V).	Voltage per mean free path (V/n).	Temperature corresponding to energy of electrons after falling through (V/n).
						°
0·46	0·9	0·09	10	870	87	670,000
0·13	2·5	0·30	8·3	2720	330	2,600,000

The average energy with which the electrons are projected through the gas probably corresponds to considerably higher temperatures than these. The upper limit to this energy is fixed by the total voltage drop. This would correspond to temperatures of 6,700,000° and 21,000,000° respectively.

The changes which give rise to the large evolution of energy in giant stars certainly take place in those of type M. For these, according to Eddington's theory of the internal constitution of stars,* the maximum temperature at the centre is 1,400,000°. For giant stars of type F, in which the changes still occur, the maximum temperature is 6,590,000°. These figures show that the energies of the electrons in the discharge tube were such that the effect might have been expected to take place.

The order of effect to be expected cannot be estimated. The energy radiated by a giant star amounts to roughly 200 ergs per second per gramme mass of star. We do not know how much of this mass is hydrogen. The mass of hydrogen present in the discharge tube of volume 70 c.c. would be 0·0000042 grm. at a pressure of 0·5 mm. of mercury. Assuming that the star consists wholly of hydrogen, and that the same figure, namely, 200 ergs per second per gramme, which applies to all giant stars, also applies to the discharge tube, the energy evolved by these changes would be 8×10^{-4} ergs per second. This would be about 10^{-11} of the heat evolved by the passage of the discharge. We see, therefore, that the negative result of this experiment does not in any way affect the hypothesis put forward by Eddington as to the origin of stellar energy.

Another point that should be mentioned is that, if the close combination of an electron and a hydrogen nucleus did occur, the energy evolved might not appear directly as heat, but might come out in the form of a very penetrating γ-ray, which would escape detection in ordinary experiments.

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* 'Zeits. für Physik,' vol. 7, p. 351 (1921).

II. EXPERIMENTAL DETAILS.

Discharge Tube and Gas Supply.

The apparatus consisted of a cylindrical discharge tube 19 cm. long and 2·5 cm. in diameter, immersed in paraffin oil contained in a large Dewar flask 8 cm. in diameter and about 30 cm. long. The electrodes were aluminium discs 2 cm. in diameter. The tube was exhausted by a Gaede rotating mercury pump. In some of the experiments the gas was absorbed, and in order to keep the conditions in the tube constant, hydrogen prepared by electrolysis of dilute sulphuric acid was allowed to leak into it through a capillary from a reservoir, the pressure in which was adjusted to a suitable value. The discharge tube itself could not be seen, and, in order to be able to observe under what conditions the discharge was running, a similar tube was connected to the tube leading to it. Arrangements were made so that this outside discharge tube could be substituted for the one inside the flask in the electrical circuit. This tube was used to measure the length of the dark space. For the same pressure, the two tubes were practically identical as regards the relation between current and voltage.

The flask was closed by a cylindrical wooden block, fitting over its mouth, which carried the discharge tube, stirrer, etc., which were inside it.

Thermostat.

Preliminary experiments showed that one of the serious difficulties of the experiment was the relatively large value of the heat losses. This was due to the fact that the power which can be put through a discharge tube is so small in proportion to the volume it occupies. If one tries to increase the power beyond certain limits, the discharge becomes very unsteady. This fact made it necessary that the heat losses should be definite and determinable. In order that this should be the case, the flask was immersed in an electrically heated thermostat with a toluene mercury control. The temperature of this thermostat was constant to less than 1/100° during an experiment. This temperature, which determined the most important heat loss, was read on a Beckmann thermometer.

The air temperature, which determines the loss through the wooden top of the flask, was read on an ordinary mercury in glass thermometer.

Measurement of Rise of Temperature.

The rise of temperature of the oil was measured by five naked copper-constantan thermo-junctions in series. The cold junctions inside a glass tube were immersed in a mixture of ice and distilled water contained in a

Dewar flask. Each lead to the junctions, both those in the oil and the cold junctions passed through a separate glass tube to ensure that no insulation leaks should occur.

The oil was stirred by a metal stirrer carried on a glass rod. The stirrer just cleared the walls of the flask, and was proved to be efficient by connecting two opposed thermo-junctions, one at the top and the other at the bottom of the flask, straight through the galvanometer. A revolution counter was attached to the stirrer.

Arrangement of Circuits.

The electrical arrangements are shown in fig. 1. The discharge tube D and a megohm, M, were connected in parallel. The milliammeters A_1 and A_2 were used to determine the currents in these two parts of the circuit. The current through the megohm determines the voltage drop over the tube. The resistances R_1 and R_2 were used to calibrate A_1 and A_2.

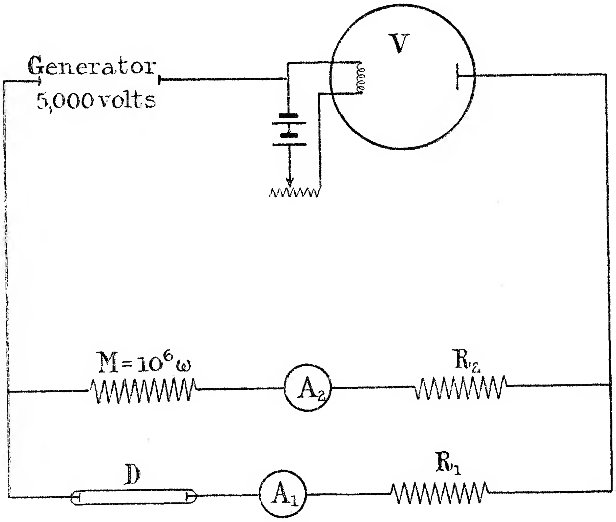

Fig. 1.

Compared to the resistance of the discharge tube, which was always greater than 1 megohm, and that of the megohm, the resistances $(R_1 + A_1)$ and $(R_2 + A_2)$ could always be neglected, since neither was greater than 300 ohms.

The source of power was an Evershed 5000 volt 15 milliampère direct current generator. The variations in the voltage of this generator were a serious source of trouble until a valve V was inserted in the circuit. I am indebted to Dr. Langmuir who suggested this important improvement. The current through the filament of this valve was so arranged that under the conditions of working the total current through the megohm and discharge tube was the saturation current of the valve. Any variations in the voltage of the generator including those small ones due to imperfect rectification were thus eliminated. The remaining cause of unsteadiness was the variation of the conditions prevailing in the discharge tube, and this was made as small as possible by the gas circulating device already described.

Insulation.

In order to ensure that insulation troubles should not arise, all the leads were suspended from glass tubes or were attached to blocks of paraffin wax or ebonite. The leads into the flask itself passed through separate glass tubes through the wooden top.

The insulation between the discharge tube circuit and the thermocouple circuit was tested by closing the latter circuit and switching on the generator. No deflection of the galvanometer could be observed.

Potentiometer.

The electromotive force developed by the thermocouples was measured on a Tinsley vernier potentiometer. This instrument was also used to measure the potential differences required for calibrations. It was not calibrated, as it had not been used since the makers had calibrated it and found it correct to at least 1 part in 10,000. A suspended coil galvanometer of the Ayrton-Mather type was used as an indicating instrument. The standard of E.M.F. consisted of two Weston cells by Tinsley.

Experimental Method.

The flask contained a heating coil of manganin wire. A measurement was made of the energy that had to be supplied to this coil to produce the same rise of temperature of the oil as was produced by the discharge tube working under known current and voltage. The electrical energies supplied in the two cases were compared. Since the thermal capacity was the same these two amounts of energy should agree, unless in the case of the hydrogen discharge changes occur which give rise to an evolution of energy. This comparative method made calibration of the thermocouples unnecessary. It is equivalent to determining the thermal capacity of the flask and its contents by means of the wire heating coil.

Measurement of Supply of Electrical Energy.

The energy supplied to the discharge tube was measured by reading the milliammeters A_1 and A_2 (fig. 1) as frequently as possible during the course of an experiment. These readings determine respectively the current through the tube and that through the megohm in parallel with the tube which gives the voltage drop over the tube. The product of the two readings was taken and the mean of two successive values of this product multiplied by the time interval between them. By adding together all the numbers so obtained we get the integrated energy supply for the whole experiment.

In the case of the wire heater, H (fig. 2), the resistance of the heater was determined, and the current through it measured by finding on the potentiometer the potential drop over the resistance R_3.

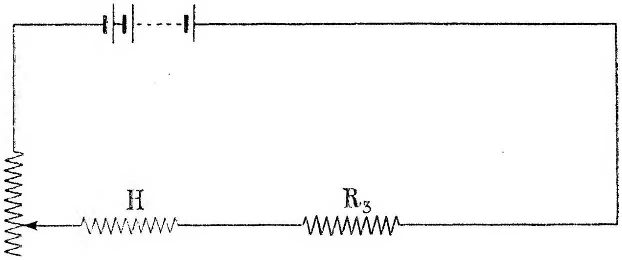

FIG. 2.

The resistances R_1 and R_2 which were of manganin and were oil immersed were used to calibrate A_1 and A_2 respectively. This calibration was effected by short-circuiting the valve V, replacing the generator by a single cell and a variable resistance, and short-circuiting the discharge tube to calibrate A_1 and the megohm to calibrate A_2. The instruments were read and the corresponding potential drops over R_1 and R_2 determined on the potentiometer.

To compare the results of heating by the wire and the tube it is necessary to know R_1, R_2, R_3, H and the megohm M in terms of one another. All these comparisons were made using the potentiometer.

$$H = \left.\begin{matrix} 10{\cdot}765 \\ 10{\cdot}764 \end{matrix}\right\} \text{mean} = 10{\cdot}76 \text{ ohms},$$

$$R_3 = \left.\begin{matrix} 0{\cdot}80133 \\ 0{\cdot}80133 \end{matrix}\right\} \text{mean} = 0{\cdot}8013 \text{ ohms}.$$

These were both compared directly with a standard 1 ohm coil by Tinsley.

$$R_1 = \left.\begin{matrix} 104{\cdot}66 \\ 104{\cdot}66 \end{matrix}\right\} \text{mean} = 104{\cdot}66 \text{ ohms},$$

$$R_2 = \left.\begin{matrix} 212{\cdot}04 \\ 212{\cdot}04 \end{matrix}\right\} \text{mean} = 212{\cdot}04 \text{ ohms}.$$

These were each determined by comparison with a 10 ohm manganin coil which was directly compared with the standard 1 ohm. Their ratio was checked at intervals. The megohm was subdivided into 20 coils of

50,000 ohms. Each of these coils was compared directly with R_2 with an accuracy of 1 part in 5000, so that if all the errors were in the same direction the maximum possible error in the sum would be 1 part in 250. The values obtained were :—

Coil.	Resistance.	Coil.	Resistance.
1	50,030	11	50,000
2	49,990	12	50,010
3	50,000	13	49,990
4	49,980	14	50,010
5	49,990	15	50,010
6	50,010	16	50,010
7	50,030	17	50,060
8	50,030	18	50,000
9	49,990	19	50,000
10	50,040	20	50,000

Total M = 1,000,180 ohms.

Carrying Capacity of Heating Coil and Megohm.

It was necessary to determine whether, under the conditions of the experiment, there was any appreciable change in the resistance of the heating coil, H, and the megohm, M, due to the heating effect of the currents passing through them. In the case of the heating coil this was tested by making a coil of the same gauge manganin wire as the coil itself was made of whose resistance was nearly the same as that of another coil of much thicker wire. The two coils were connected in series and immersed in oil. The ratio of the potential drops over them was determined on the potentiometer for different currents flowing through them. It was found when the current was varied from 0·2 to 0·72 ampères, that the ratio of the resistances did not change by 1 part in 1000. This shows that over this range of currents the change in resistance of the heating coil is inappreciable. This is practically the same as the range of currents used in the experiments, which was 0·1 to 0·8 ampères.

To test the megohm it was arranged so that one of the twenty coils was in the arm AD of the Wheatstone's bridge, fig. 3. The other nineteen coils were in the arm AB. The arms DC and BC were 100 and 1900 ohms respectively, and were of much heavier wire than the megohm. The change in the ratio of the resistances of AB and AD was less than 1 part in 1000 when the potential applied to the bridge

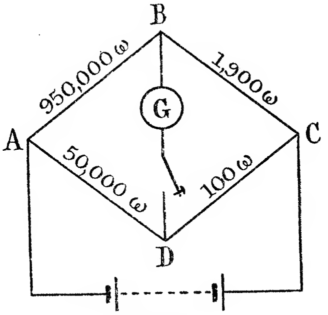

Fig. 3.

changed from 20 to 200 volts. This means that the resistance of AD did not change appreciably when the current through it was varied from 0·4 to 4 milliampères, since the change of current in the arm AB was inappreciable compared to this change. The maximum current passing through the megohm in the experiments performed was 3 milliampères.

Calculation of Energy Supply.

The method of determining the energy supply will be illustrated by giving, in tabular form, the calculations for a typical experiment. The second column contains the product of the observed deflections of the ammeters A_1 and A_2. The calibration factors are applied later.

Time.	Product of readings of A_1 and A_2.	Interval (T).	Mean product during interval (P).	P × T.
′ ″		″		
12 22	238·8			
14 37	236·4	(135)42	237·6	0·998 × 10^4
16 20	232·4	103	234·4	2·414
18 50	227·3	150	229·9	3·448
20 37	224·7	107	226·0	2·418
22 40	222·4	123	223·5	2·749
24 45	219·4	125	220·9	2·761
27 10	216·1	145	217·8	3·159
29 17	213·3	127	214·7	2·727
31 47	208·3	150	210·8	3·161
33 37	206·3	110	207·3	2·280
36 35	201·5	178	203·9	3·629
38 20	199·4	105	200·5	2·105
40 47	194·4	(147)99	196·9	1·949

Sum = 33·798 × 10^4,
$\doteq$ 3·380 × 10^5.

It will be noticed that in the cases of the first and last interval only part of the interval has been used as multiplier. This is because the rise of temperature was measured from 13 minutes 55 seconds to 39 minutes 59 seconds.

To reduce this result to watt-seconds we used the calibration factors of A_1, A_2. If e_1 and e_2 respectively are the voltage drops, as measured on the potentiometer over R_1 and R_2, corresponding to unit deflection of A_1 and A_2, we have the energy supply in watt seconds

$$= \frac{3{\cdot}380 \times 10^5 \times e_1 \times e_2}{R_1 \times R_2} \times 10^6.$$

It was found that for the shunts used in this experiment

$$e_1 = 0{\cdot}02811\,; \qquad e_2 = 0{\cdot}01711.$$

Inserting these values, and the values of R_1 and R_2, namely, 104·7 and 212·0, we obtain

$$\text{Energy supply} = \frac{3{\cdot}380 \times 10^5 \times 0{\cdot}02811 \times 0{\cdot}01711}{104{\cdot}7 \times 212{\cdot}0} \times 10^6$$

$$= 7{\cdot}324 \times 10^3 \text{ watt-seconds.}$$

Correction for Losses of Heat.

Changes in the temperature of the oil in the calorimeter may be caused by four factors other than the heat generated by the current in the discharge tube:—

(1) Communication of heat between the oil in the flask and the thermostat.

(2) Conduction through the top of the flask to the air.

(3) Heat generated by stirring.

(4) The temperature of the discharge tube will be above that of the oil in the flask, because heat is flowing from it to the oil. The excess temperature of the tube will depend on the power supply, since this determines the rate of flow of heat from the tube to the oil. If the power supply changes the excess temperature of the discharge tube will change. In other words, the amount of heat energy stored in the tube will change. This will be equivalent to a loss or gain of heat.

The corrections for the first three losses can be considered together as they were determined by the same method. The rate of change of thermocouple reading, with no current through the discharge tube, was plotted against thermocouple reading. This curve, which was a straight line, was drawn for the following standard conditions:—

Reading of Beckmann thermometer ... = 3·35 (22·7° C.)
Air temperature .... = 16°
Rate of stirring .... = 4 revolutions per second.

If all the experiments had been done under these standard conditions, the change in thermocouple reading due to losses during the course of an experiment could have been obtained from the graph. Actually, corrections had to be made to the rates of change determined from the graph for variations from the standard conditions.

The temperature of the thermostat was not the same throughout the whole series of experiments, owing to the fact that the regulator had to be readjusted several times. Correction was made for these changes by determining the change in thermocouple E.M.F. corresponding to 1/100°. This was found to be $0{\cdot}00206 \times 10^{-3}$ volts per 1/100°. Then for a thermostat temperature above 3·35, for example, 3·43 or 8/100° above, we subtract $8 \times 0{\cdot}00206 \times 10^{-3}$ from all the thermocouple readings, and read the rates of changes from the graph for the readings so obtained. Since the rate of change depends only on the difference between the oil and thermostat temperature, the values of the rate of change so obtained will be corrected for variations in the thermostat temperature from the standard. In other words, raising the thermostat temperature produces the same effect as lowering the oil temperature by the same amount.

Curves of rate of change of thermocouple reading plotted against thermocouple reading were drawn for different rates of stirring and for different air temperatures. By comparing curves in which (*a*) the variation of rate of stirring was large and that of air temperature small, (*b*) the variation of air temperature was large and that of rate of stirring small, it was possible to separate the two effects. The two groups of experiments will be considered separately :—

(*a*) In the experiments to determine the effect of changes in the rate of stirring on the rate of change of thermocouple reading, slight variations of the air temperature occurred. To correct for these variations, an approximate value for the effect of change of air temperature was required. This was obtained from two sets of readings at 22·05° C. and 16°–17° C. The approximate value was that 1° change in the air temperature produces a change of $0·0000048 \times 10^{-3}$ volts per second in the rate of change of thermocouple readings. This was almost identical with the final corrected value 0·0000049. Applying this correction, the following results were obtained :—

Change in rate of stirring. δs.	Corresponding change in rate of change of thermo-couple reading. Δ.	$\Delta/\delta s$.
1·37 revs. per sec.	$0·0000062 \times 10^{-3}$ volts per sec.	0·0000045
1·70 ,, ,,	0·0000075 ,, ,,	0·0000044
2·95 ,, ,,	0·0000096 ,, ,,	0·0000033
2·64 ,, ,,	0·0000157 ,, ,,	0·0000059

The numbers in the last column show that the correction to the rate of change for variations in the stirring may be taken as proportional to the variation in the rate of stirring. The mean of the numbers in the last column is 0·0000045. The error in this mean is certainly less than 0·000001. The two largest corrections to the rise in temperature of the oil due to variation in the rate of stirring from 4 revolutions per second amounted to 3 and 2·4 per cent. of the total rise. An error of 0·000001 in this figure would produce errors in the final results of 0·7 and 0·5 per cent. respectively. The actual errors due to the uncertainty of this correction will therefore be less than these.

(*b*) In the experiments to determine the effect of changes in the air temperature variations in the rate of stirring were corrected for among the figures just obtained. It was found that a change of 1° in the air temperature produced a change in the rate of change of thermocouple reading of $0{\cdot}0000049 \times 10^{-3}$ volts per second. This was the mean of twenty-three observations. The average departure from the mean was 0·0000005. The two largest corrections to the rise in temperature of the oil due to variations of the air temperature from 16° C. amount to 3 per cent., and a little less than 3 per cent. of the total rise. An error of 0·0000005 in the figure used to calculate these corrections would produce errors in the final result of 0·3 per cent. The uncertainty of this correction is less than this mean departure of the individual values.

Using these two correcting factors, all the observed rates of change were corrected to what they would have been if the rate of stirring had been 4 revolutions per second and the air temperature 16° C. The values so obtained were plotted against thermocouple readings, corrected, as explained above, for variations in the temperature of the thermostat from 3·35. This gave the heat-loss curve for the standard conditions. The points were found to lie on a straight line over the range of thermocouple readings $3{\cdot}4 \times 10^{-3}$ volts to $5{\cdot}9 \times 10^{-3}$ volts, that is, from 17° to 29° C.

We now consider the correction for changes in the amount of heat stored in the discharge tube due to changes in the power supply. This was determined by finding the amount of heat liberated when the power supply was changed by a known amount. To do this the following experiments were carried out: The scale of the galvanometer was calibrated. The discharge tube was run, the power supply being 6·86 watts. The current was shut off at a known instant, and the reading of the galvanometer scale noted at this time. Readings of the scale were taken at short intervals after this, and it was found that the oil temperature continued to rise after the power had been shut off. The deflections were reduced to thermocouple E.M.F.'s, and the values corrected for losses in the interval with the following results:—

Time in seconds from shutting off power.	Thermocouple E.M.F. (volts).
0	$5{\cdot}1592 \times 10^{-3}$
10	5·1652
25	5·1725
35	5·1745
50	5·1769
65	5·1785
80	5·1793
90	5·1798
100	5·1814

We see that when the power supply decreases by 6·86 watts the heat energy which is stored in the tube decreases, and that it causes a rise of $0·022 \times 10^{-3}$ volts in the thermocouple E.M.F. when it is distributed.

The correction may be taken as proportional to the change in power supply and to be equal to $0·00031 \times 10^{-3}$ volts for a change of 1 watt.

In applying this correction to the measured rise of temperature it is necessary to consider only the difference between the initial and final values of the power supplied.

Calculation of Heat-Loss Corrections.

The application of these corrections will be shown by taking a typical experiment and giving the complete reduction of the results.

Adding the figures in the last column (see Table facing) we obtain the total change of thermocouple E.M.F. due to the passage of heat through the walls and top of the flask $= -0·0455 \times 10^{-3}$ volts.

We now correct for stirring, remembering that the graph has been drawn for stirring at the rate of four revolutions per second :—

Interval.	Mean rate of revolution of stirrer in interval.	Rate of change of thermo. E.M.F. due to difference of rate of stirring from 4 revs. per sec.	Change of thermo. E.M.F. in interval due to excess stirring.
257	4 ·50	+0 ·0000023	+0 ·0006
206	4 ·70	32	+0 ·0007
267	5 ·01	45	+0 ·0012
292	4 ·27	12	+0 ·0003
290	4 ·45	20	+0 ·0006
252	5 ·06	48	+0 ·0012

Adding the figures in the last column we obtain the total change of thermocouple E.M.F. due to excess rate of stirring $= +0·0046 \times 10^{-3}$ volts.

The power at the end of the experiment was less than that at the beginning by 0·95 watts. We saw that a decrease of 1 watt produces an increase in the temperature of the oil as measured by the thermocouple E.M.F. of $0·0031 \times 10^{-3}$ volts. We have, therefore, the change of thermocouple E.M.F. due to change in the supply of power $= +0·0029 \times 10^{-3}$ volts.

Time.	Thermo-couple E.M.F. (volts × 10^{-3}).	Air temperature.	Thermostat temperature, Beckmann thermometer.	Interval T (seconds).	Mean thermo. E.M.F. during interval.	Mean thermo-couple E.M.F. corrected for variation of thermostat temperature from 3·35.	Rate of change of thermocouple from graph drawn for standard conditions.	Correction to rate of change for variation of air temperature from 16° C.	Rate of change corrected for variation of air temperature.	Change due to loss to thermostat and air in interval T (volts × 10^{-3}).
′ ″										
13 55	4·2462	16·55	3·395							
				257	4·3113	4·3007	+0·0000140	+0·0000022	+0·0000162	+0·0041
18 12	4·3764	16·35	3·39							
				206	4·4264	4·4182	−0·0000038	18	−0·0000020	−0·0004
21 38	4·4765	16·40	3·39							
				267	4·5366	4·5284	−0·0000208	19	−0·0000189	−0·0050
26 5	4·5967	16·40	3·39							
				292	4·6568	4·6486	−0·0000395	24	−0·0000371	−0·0108
30 57	4·7169	16·60	3·39							
				290	4·7720	4·7638	−0·0000571	25	−0·0000546	−0·0158
35 47	4·8270	16·45	3·39							
				252	4·8721	4·8639	−0·0000725	25	−0·0000700	−0·0176
39 59	4·9171	16·60	3·39							

Collecting these results we have

Change of thermo. E.M.F. from 13′ 55″ to 39′ 59″ $= 4{\cdot}9171 - 4{\cdot}2462$ .... $= +0{\cdot}6709 \times 10^{-3}$ volts.
Change due to passage of heat through walls and top of flask .... $= -0{\cdot}0455 \times 10^{-3}$ „
Change due to excess stirring above 4 revolutions per second .... $= +0{\cdot}0046 + 10^{-3}$ „
Change due to decrease of power supply $= +0{\cdot}0029 \times 10^{-3}$ „

We therefore have for $\delta\theta$ the change of thermocouple E.M.F. that would have been produced by the heat generated in the discharge tube alone.

$$\delta\theta = (0{\cdot}6709 + 0{\cdot}0455 - 0{\cdot}0046 - 0{\cdot}0029) \times 10^{-3} \text{ volts.}$$
$$= 0{\cdot}7089 \times 10^{-3} \text{ volts.}$$

Summary of Results.

In the following Tables the results which were obtained are summarised. The first Table refers to the experiments with the hydrogen discharge tube. The numbers in the sixth column, headed $\delta\theta$, give the change of thermocouple E.M.F. corrected for all losses. Those in the second last column give the total energy supplied to produce this change. In the last column is given the energy required to produce a change of 10^{-3} volts in the thermocouple E.M.F. In deducing this figure we assume that the relation between thermo. E.M.F. and temperature is approximately linear over the range of temperatures used. This range was 3° to 5°. We also assume that the specific heat of the oil is constant.

It will be noticed that there is no tendency for the figures in the last column to vary in a systematic way with the voltage. Their mean is $1{\cdot}035 \times 10^{-4}$ watt-seconds. The average departure from the mean is $0{\cdot}004 \times 10^{-4}$. It should be mentioned that there is no tendency for the results to vary in a systematic way with the time of the experiment, the air temperature, or the stirring. This shows that the corrections made for losses are sufficiently accurate.

In the case of the experiments in which the oil was heated by the wire heater, the following results were obtained for the energy required to produce a change of 10^{-3} volts in the thermocouple reading:—

Watt-seconds.	Watt-seconds.
$1{\cdot}043 \times 10^{4}$	$1{\cdot}040 \times 10^{4}$
$1{\cdot}039 \times 10^{4}$	$1{\cdot}041 \times 10^{4}$
$1{\cdot}034 \times 10^{4}$	$1{\cdot}029 \times 10^{4}$

Hydrogen Experiments.

Time.	Measured change of thermo. E.M.F. (volts × 10^{-3}).	Change due to		Decrease of power supply (volts × 10^{-3}).	$\delta\theta$ (volts × 10^{-3}).	Mean voltage.	Mean current (× 10^{-3} ampères).	Energy supply (watt-secs.).	Energy supply.
		Conduction of heat (volts × 10^{-3}).	Excess stirring (volts × 10^{-3}).						Corrected change of temperature.
″ 2675	0 ·6916	−0 ·0496	+0 ·0012	+0 ·0025	0 ·7375	866	3 ·34	7 ·696 × 10^3	1 ·043 × 10^{-4}
2185	0 ·6506	−0 ·0166	−0 ·0004	+0 ·0028	0 ·6648	979	3 ·18	6 ·873	1 ·035
1856	0 ·7391	−0 ·0509	+0 ·0110	+0 ·0041	0 ·7749	1150	3 ·71	8 ·063	1 ·040
1564	0 ·6709	−0 ·0455	+0 ·0046	+0 ·0029	0 ·7089	1363	3 ·45	7 ·324	1 ·033
1293	0 ·8526	−0 ·0248	+0 ·0064	+0 ·0028	0 ·8682	1617	4 ·30	8 ·991	1 ·035
1511	0 ·8600	−0 ·0259	+0 ·0110	+0 ·0034	0 ·8715	1652	3 ·61	8 ·999	1 ·033
1122	0 ·8635	−0 ·0224	+0 ·0064	+0 ·0029	0 ·8766	1670	4 ·39	9 ·053	1 ·032
1644	0 ·8891	−0 ·0258	+0 ·0125	+0 ·0137	0 ·8987	1713	3 ·30	9 ·272	1 ·033
2420	0 ·7628	−0 ·0379	+0 ·0177	—	0 ·7830	2050	1 ·64	8 ·058	1 ·029
2170	0 ·7712	−0 ·0660	+0 ·0255	+0 ·0009	0 ·8108	2260	1 ·72	8 ·356	1 ·030
1473	0 ·6203	−0 ·0275	−0 ·0022	—	0 ·6510	2716	1 ·70	6 ·798	1 ·044

These experiments were done under a variety of conditions as regards duration of experiment and air temperature. The results again did not show any systematic variation. Their mean is $1{\cdot}038 \times 10^4$ watt-seconds and the average departure is 0·004 watt-seconds.

It will be seen that the energy supply to produce the same rise of temperature is the same in the two cases to within the error of the experiment, the two values being $1{\cdot}035 \times 10^4$ watt seconds for the hydrogen discharge tube and $1{\cdot}038 \times 10^4$ watt-seconds for the wire heater. These values agreed with the value obtained in two or three experiments in which the tube was filled with air. This value was $1{\cdot}036 \times 10^4$ watt-seconds.

Discussion of Results.

It is interesting to note that these experiments prove that the conservation of energy holds in the energy changes in a discharge tube.

It would probably be possible to increase the accuracy of the heat measurements to 1 part in 500. This, however, would be extremely difficult, as will be seen by considering the very unfavourable conditions as regards quantity of heat per unit volume and remembering that under the most favourable conditions calorimetric measurements seldom attain an accuracy greater than 1 part in 1000. It would not be worth while doing this until a perfectly steady discharge could be obtained. At present this appears to be impossible owing to the large number of variables involved. In the present work two experiments had to be rejected on account of this unsteadiness. In many other cases conditions were so bad that it was impossible to obtain any measurements at all.

In conclusion it is a pleasure to thank Sir Ernest Rutherford for suggesting the experiment and for the interest he has taken in its progress. My thanks are also due to Mr. E. A. Milne of Trinity College for supplying me with the astrophysical data.